Score: ___ / 100 Name: Key Instructor: ____________

Midterm Exam Part I (Calculator Section) MA1161 Spring 2006

Show sufficient work to justify all answers.

1. The temperature, T, of a hot object put in the refrigerator is given by

$$T = f(t) = 195 - \frac{3t}{50},$$

where t is the time in seconds since the object was put in to refrigerator.

(a) Find a formula for the inverse function $f^{-1}(T)$.

$T = 195 - \frac{3t}{50}$

$-50(T-195) = 3t$

$-\frac{50}{3}(T-195)$

$f^{-1}(T) = -\frac{50}{3}T + 3250$

$f^{-1}(T) =$ $-\frac{50}{3}(T-195)$ [3]

(b) Evaluate and give the practical meaning $f^{-1}(32)$.

an object at a temperature of $32°$ has been in the oven 2716.7 sec (or 45.28 min)

$f^{-1}(32) =$ 2716.7 sec (45.28 min) [3]

2. Give a formula for a rational function which has x-intercepts at $x = 2$ and $x = -3$ and a vertical asymptote at $x = 1$.

$\frac{(x-2)(x+3)}{(x-1)}$

$y =$ $\frac{(x-2)(x+3)}{(x-1)}$ [4]

3. Suppose that after 5 years, 60% of a radioactive substance is left.

(a) Assuming the substance decays at a continuous rate, fill in the following table, where P is the percentage of the original amount left after t years.

t (years)	0	5	10	15
P (percent)	100	60	36	$108/5 = 21.6$

ratio: $\frac{60}{100} = \frac{3}{5}$ $\quad \frac{x}{60} = \frac{3}{5}$ $\quad \frac{x}{36} = \frac{3}{5}$

[2]

(b) Find a possible formula for $P(t)$.

$P(t) = P_0 e^{kt}$

$P_0 = 100$ $\quad k = \frac{\ln\left(\frac{3}{5}\right)}{5} \approx$

$60 = 100e^{k5}$

$\frac{3}{5} = e^{5k}$

$\ln\left(\frac{3}{5}\right) = 5k$

$P(t) = 100e^{\frac{(\ln\frac{3}{5})t}{5}} \approx 100e^{-.1022t}$ [4]

(c) Use your formula to find the half-life of the substance.

$\frac{1}{2} = e^{\frac{(\ln\frac{3}{5})}{5}t}$ (1)

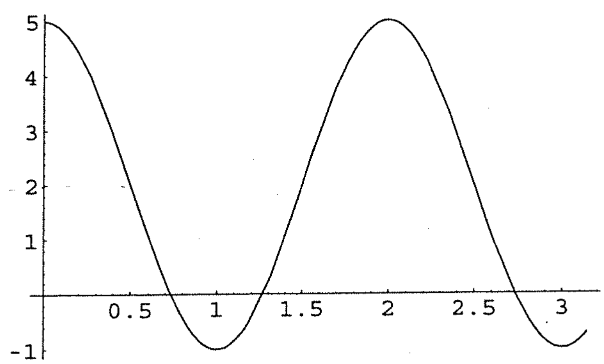
$\ln\frac{1}{2} = \frac{\ln\frac{3}{5}}{5}t$

$\frac{5\ln\frac{1}{2}}{\ln 3/5} = t$

(2)

Half-Life = $\frac{5\ln\frac{1}{2}}{\ln 3/5} \approx 6.78$ yrs (1) [4]

4. Find a possible formula for the graph of the sinusoidal function shown below:



$|A| = 3$

period = 2

$y = C + A\cos(Bx)$

$\frac{2\pi}{B} = 2$

$B = \pi$

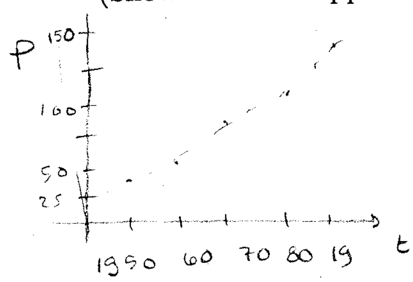
$y = 2 + 3\cos(\pi x)$ [4]

$y = 2 + 3\sin[\pi(x - 0.5)]$

5. The table gives the population $P = f(t)$, in millions, in some country in the year t.

t	1950	1960	1970	1980	1990
P (Population in millions)	40	52.8	74.2	111.8	144.1

(a) Does $f'(t)$ appear to be positive or negative during the years between 1950 and 1990? (Show work to support your answer)



slopes are positive

$f'(t)$ is positive [3]

(b) Does $f''(t)$ appear to be positive or negative during the years between 1950 and 1990? (Show work to support your answer)

rate of change 1950-1960:

$$\frac{52.8-40}{10} = 1.28$$

rate of change increasing

1980-1990:

$$\frac{144.1-111.8}{10} = 3.23$$

$f''(t)$ is positive [3]

(c) Estimate $f'(1975)$. (Include units)

ave. rate. of change

1970 -1980:

$$\frac{111.8-74.2}{10} = 3.76$$

$f'(1975) =$ 3.76 mil. people/year [3]

6. At a time t seconds after it is thrown in the air, a tomato is at a height of $h(t) = -4.9t^2 + 25t + 3$ meters.

(a) What is the average velocity of the tomato during the first 2 seconds? (Include units)

$$\frac{h(2)-h(0)}{2-0} = \frac{-4.9(4)+25(2)+3-[0+0+3]}{2} = \frac{30.4}{2}$$

(1) (1)

Average Velocity = 15.2 m/s [4]

(b) Find the exact instantaneous velocity of the tomato at $t = 2$ seconds. (Include units)

$h'(t) = -9.8t + 25$ (2)

$h'(2) = 5.4$

Instantaneous Velocity = 5.4 m/s [4]

(c) What is the acceleration at $t = 2$ seconds? (Include units)

$h''(t) = -9.8$ (2)

$h''(2) = -9.8$

Acceleration = $-9.8\ m/s^2$ [4]

7. Let $f(x) = x^2 + 3x + 2$. Find $f'(1)$ algebraically using the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^2 + 3(1+h) + 2 - [1+3+2]}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 + 3 + 3h + 2 - 6}{h}$$

$$= \lim_{h \to 0} \frac{5h + h^2}{h}$$

$$= \lim_{h \to 0} 5 + h = 5$$

$f'(1) =$ 5 [6]

8. An economist is interested in how the price of a certain commodity affects its sales. Suppose that at a price of p dollars, a quantity q of the commodity is sold.

(a) If $q = f(p)$, explain in economic terms the meaning of the statements $f(10) = 240,000$ and $f'(10) = -29000$.

$f(10) = 240,000$ means if the price is \$10 you can sell 240,000 items [4]

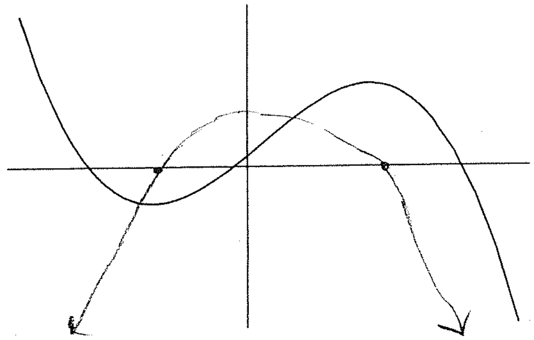
$f'(10) = -29000$ means for every dollar over \$10 you charge the number of items sold will drop by 29,000 [4]

(b) Using this information, estimate $f(12)$.

$$f(12) \approx 240,000 - 2(29000)$$

$f(12) =$ 182,000 [4]

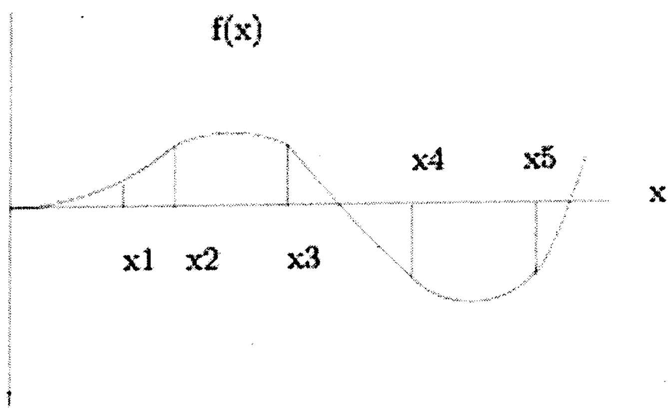
9. The graph of $f(x)$ is shown below. Sketch a possible graph of $f'(x)$ on the same set of axes.



1 pt each zero
1 pt each +/− in correct intervals

[5]

10. At which of the marked x-values in the figure can the following statements be true?.



(a) $f(x) < 0$ — (a) X_4, X_5 [2]

(b) $f'(x) < 0$ — (b) X_3, X_4 [2]

(c) $f(x)$ is decreasing — (c) X_3, X_4 [2]

(d) $f'(x)$ is decreasing — (d) X_2, X_3 [2]

(e) Slope of $f(x)$ is positive — (e) X_1, X_2, X_5 [2]

(f) Slope of $f(x)$ is increasing — (f) X_1, X_4, X_5 [2]

[12]

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Midterm Exam Part II (Non-Calculator Section) MA1161 Spring 2006

Show sufficient work to justify all answers.

1. (a) Find the derivative of $y = 4t^2 + \frac{13}{\sqrt{t}} - \frac{2}{t^2}$.

$y = 4t^2 + 13t^{-\frac{1}{2}} - 2t^{-2}$

$y' = 8t - \frac{13}{2}t^{-\frac{3}{2}} + 4t^{-3}$

$y' = 8t - \frac{13}{2t^{3/2}} + \frac{4}{t^3}$ [4]

or $y' = 8t - \frac{13}{2}t^{-3/2} + 4t^{-3}$

(b) Find $\frac{dy}{dx}$ if $y = 2^x$.

$\frac{d(2^x)}{dx} = \ln(2)2^x$

$\frac{dy}{dx} = (\ln 2)2^x$ [4]

(c) If $g(x) = x^2 \cdot e^x$, find $g'(x)$.

$g'(x) = \left(\frac{d}{dx}x^2\right)e^x + x^2\left(\frac{d}{dx}e^x\right)$

$= 2xe^x + x^2e^x$

$g'(x) = (2x + x^2)e^x$ [4]

or $g'(x) = 2xe^x + x^2e^x$

2. (a) If $f(x) = 2x^2 + x$, find $f'(2)$.

$f'(x) = 4x + 1$

$f'(2) = 4(2) + 1 = 9$

$f'(2) =$ 9 [4]

(b) Find the equation of the tangent line to $f(x)$ at $x = 2$.

$f'(2) = 9$ so slope is 9

$f(2) = 2(4) + 2 = 10$ so $(2, 10)$ is a point on the line

The equation of the tangent line is $(y - 10) = 9(x - 2)$ [4]

or $y = 9x - 8$